Assiut University
Mechanical Engineering Department
Gas Dynamics

Dec. 14th, 2011
First midterm
Exam time: 75 minutes
Total marks: 20

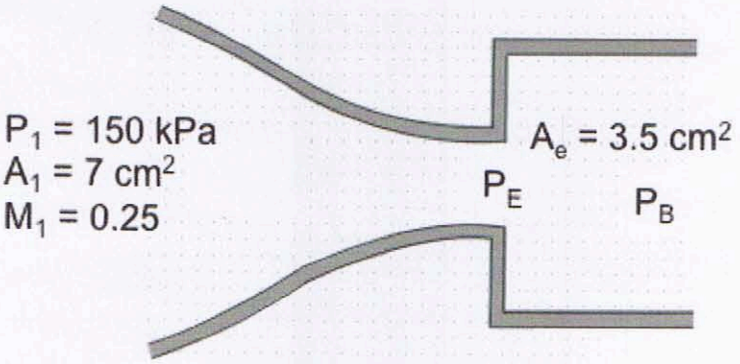
You can use Gas Dynamics tables

Solve all the following questions

Question #1 (6 marks)

Air flows into a converging nozzle as shown in the figure below.

1. Is the flow chocked? (2 marks)
2. Find the exit pressure (P_E) and the back pressure (P_B). (2 marks)
3. Find the value of P_B to cause chocking. (2 marks)



1) at $M_1 = 0.25 \longrightarrow \frac{A_1}{A^*} = 2.4$ & $\frac{P_1}{P_o} = 0.9574$

$\therefore \frac{A_e}{A^*} = \frac{A_e}{A_1}\frac{A_1}{A^*} = \frac{3.5}{7} \times 2.4 = 1.2$ 1

$\therefore \boxed{M_e = 0.59 < 1}$ 1 $\longrightarrow$ flow is not chocked

at $M_e = 0.59 \longrightarrow \frac{P_e}{P_o} = 0.7901$ 1

$\boxed{6}$

2) $\therefore P_e = P_1 \frac{P_o}{P_1} \times \frac{P_e}{P_o}$

$= 150 \times \frac{1}{0.9574} \times 0.7901 = \boxed{123.8 \text{ kPa}}$ 1

Since flow is not chocked $P_B = P_e = \boxed{123.8 \text{ kPa}}$ 1

3) at Chocking ; $M_e = 1 \longrightarrow \frac{P_E}{P_o} = 0.52828$

$\therefore P_B = P_E = P_1 \frac{P_o}{P_1} \frac{P_E}{P_o}$

$= 150 \times \frac{1}{0.9574} \times 0.52828 = \boxed{82.8 \text{ kPa}}$ 1

Question #2 (5marks)

At the entrance of an isentropic converging-diverging nozzle, $V_1 = 100$ m/s, $P_1 = 150$ kPa, and $T_1 = 300$ K, if the discharge flow is supersonic and the throat area is 0.1 m^2, determine the mass flow rate. Flowing gas is air.

$$M_1 = \frac{V_1}{\sqrt{kRT}}$$

$$= \frac{100}{\sqrt{1.4 \times 287 \times 300}} = 0.29 \longrightarrow \frac{P_1}{P_o} = 0.9432 \quad (\quad \frac{T_1}{T_o} = 0.9835$$

$$\frac{\rho_1}{\rho_o} = 0.959 \quad (\quad \frac{A_1}{A^*} = 2.098$$

$$P_o = P_1 \times \frac{P_o}{P_1}$$

$$= 150 \times \frac{1}{0.9432} = 159 \text{ kPa}$$

$$T_o = T_1 \times \frac{T_o}{T_1}$$

$$= 300 \times \frac{1}{0.9835} = 305 \text{ K}$$

$\because$ Since flow is supersonic $\longrightarrow$ Nozzle is chocked

$$\therefore \dot{m} = 0.6847 \frac{P_o}{\sqrt{R T_o}} A_{th}$$

$$= 0.6847 \frac{159 \times 10^3}{\sqrt{287 \times 305}} \times 0.1 = \boxed{36.8 \text{ kg/s}}$$

* Another solution:

$$\rho_1 = \frac{P_1}{RT_1} = \frac{150 \times 10^3}{287 \times 300} = 1.742 \text{ kg/m}^3$$

$$A_1 = A_{th} \frac{A_1}{A^*} = 0.1 \times 2.098 = 0.2098 \text{ m}^2$$

$$\therefore \dot{m} = \rho_1 V_1 A_1$$

$$= 1.742 \times 100 \times 0.2098 = \boxed{36.55 \text{ kg/s}}$$

Question #3 (4 marks)

A converging-diverging nozzle with an area ratio of (A_e/A_{th} = 3) is working at off-design conditions. The exit Mach number is 0.4 Find the pressure ratio (P_B/P_0).

at $M_e = 0.4 \longrightarrow \frac{P_e}{P_{oy}} = 0.8956$

$\frac{A_e}{A^*_y} = 1.59$

$$\frac{P_B}{P_{ox}} = \frac{P_B}{P_{oy}} \frac{P_{oy}}{P_{ox}}$$

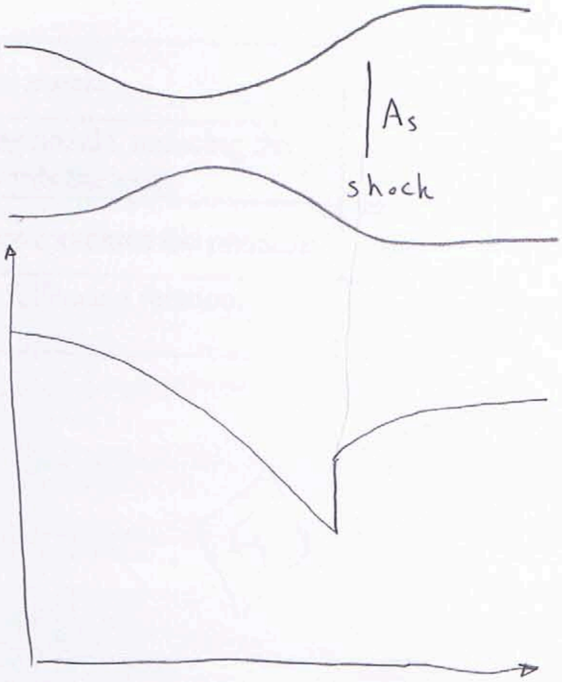
$$= \frac{P_B}{P_{oy}} \frac{A^*_x}{A^*_y}$$

$$= \frac{P_B}{P_{oy}} \frac{A^*_x}{A_e} \frac{A_e}{A^*_y}$$

$$= \frac{P_B}{P_{oy}} \frac{A_e}{A^*_y} \frac{A^*_x}{A_e}$$

$$= 0.8956 \times 1.59 \times \frac{1}{3}$$

$$= \boxed{0.475}$$



$\boxed{4}$

Question #4 (5 marks)

State whether the following statements are true or false:

1. Normal shocks increase the stagnation pressure.	X	1
2. Stagnation temperature decreases across a normal shock.	X	1
3. If a normal shock exists in a converging-diverging nozzle, reducing the back pressure moves the shock downstream (towards the exit).	✓	1
4. For isentropic supersonic flow, decreasing the area increases the pressure.	✓	1
5. The angle of a Mach cone can be found from the following relation: $\sin(\alpha) = \sin(1/M)$	X	1

$\boxed{5}$