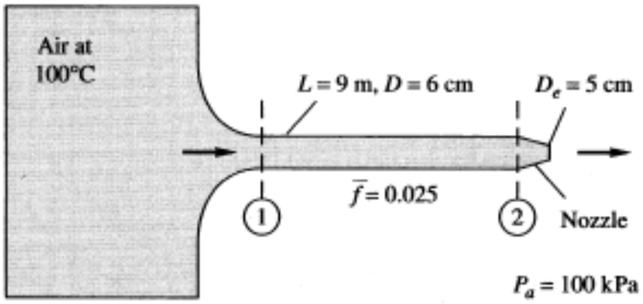
Gas Dynamics

Assignment #4: Adiabatic flow in pipes with friction and Oblique shocks

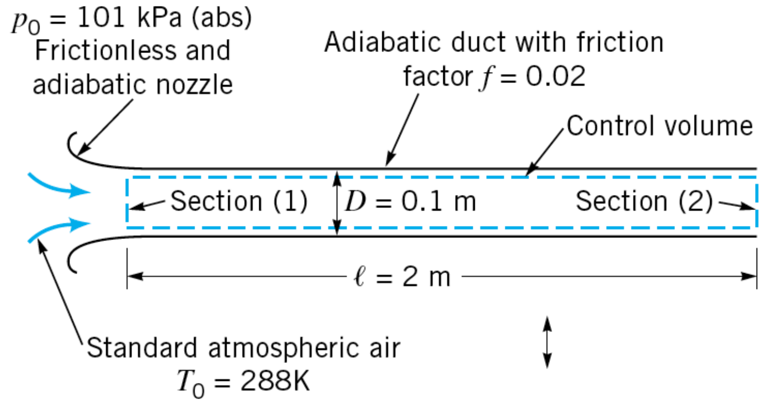
1. Fill the following table for change in flow properties inside an adiabatic duct with friction:

Property	Subsonic flow (M<1)	Supersonic flow (M>1)
Static pressure (p)	Decrease	Increase
Static temperature		
Density		
Velocity		
Mach number		
Stagnation pressure		
Stagnation temperature		
4fl/D		
Entropy (s)		

2. Air enters a 3-cm diameter pipe 15 m long at V1 = 73 m/s, p1 = 550 kPa, and T1 = 60°C. The friction factor is *4f* = 0.018. Compute V2, p2, T2, and p02 at the end of the pipe. How much additional pipe length would cause the exit flow to be sonic?
3. Air enters an adiabatic duct of L/D = 40 at V1 = 170 m/s and T1 = 300 K. The flow at the exit is choked. What is the average friction factor in the duct?
4. Air, supplied at p0 = 700 kPa and T0 = 330 K, flows through a converging nozzle into a pipe of 2.5-cm diameter which exits to a near vacuum. If *4f* = 0.022 , what will be the mass flow through the pipe if its length is (a) 0 m, (b) 1 m, and (c) 10 m?
5. Air flows steadily from a tank through the pipe in the Fig. There is a converging nozzle on the end. If the mass flow is 3 kg/s and the flow is choked, estimate (a) the Mach number at section 1; and (b) the pressure in the tank.

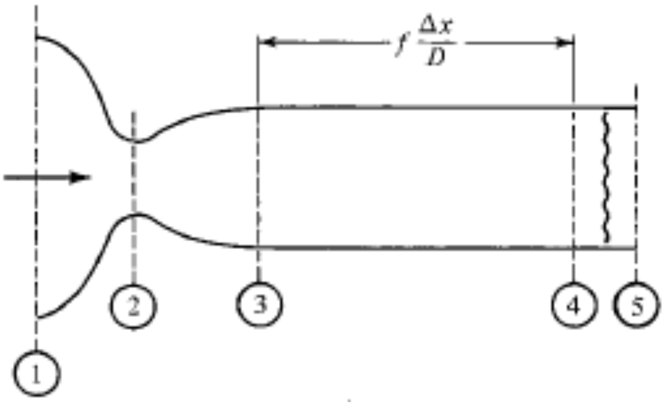


6. In the previous problem, let the tank pressure be 700 kPa, and let the nozzle be *choked*. Determine (a) Ma_2; and (b) the mass flow. Keep $T_o = 100°C$.
7. Air enters a 5-cm-diameter pipe at p1 = 200 kPa and T1 = 350 kPa. The downstream receiver pressure is 74 kPa and the friction factor is 4f = 0.02. If the exit is choked, what is (a) the length of the pipe, and (b) the mass flow? (c) If p1, T1 and preceiver stay the same, what pipe length will cause the mass flow to increase by 50% over (b)? Hint: In (c) the exit pressure does not equal receiver pressure.
8. Air, supplied by Standard atmospheric air (T_0 = 288 K, P_0 = 101 kPa) is drawn steadily through a frictionless, adiabatic converging nozzle into an adiabatic, constant-area duct as shown in the Fig. The duct is 2-m long and has an inside diameter of 0.1 m. The average friction factor for the duct is estimated as being equal to (*4f* = 0.02). What is the maximum mass flow rate through the duct? For this maximum flow rate, determine the values of static temperature, static pressure, stagnation temperature, stagnation pressure, and velocity at the inlet and exit of the duct.



9. A large chamber contains air at a temperature of 300 K and a pressure of 8 bar. The air enters a converging–diverging nozzle with an area ratio of 2.4. A constant-area duct is attached to the nozzle and a normal shock stands at the exit plane. Back pressure is 3 bar abs. Assume the entire system to be adiabatic and neglect friction in the nozzle. Compute the $4fL/D$ for the duct.

   Important hint: Normal shocks doe not change P^* (i.e. P^* is the same before and after the shock).



10. Conditions before an oblique shock are $T_1 = 40°C$, $p_1 = 1.2$ bar, and $M_1 = 3.0$. The shock is observed at $\theta = 45°$ to the approaching air flow.**(a)** Determine the Mach number and flow direction (δ) after the shock. **(b)** What are the temperature and pressure after the shock?

11. An oblique shock forms in air at an angle of $\theta = 30°$. Before passing through the shock, the air has a temperature of 288 K, a pressure of 69 kPa, and is traveling at $M = 2.6$. **(a)** Compute the normal and tangential velocity components before and after the shock. **(b)** Determine the temperature and pressure after the shock. **(c)** What is the deflection angle?

Solution of assignment #3

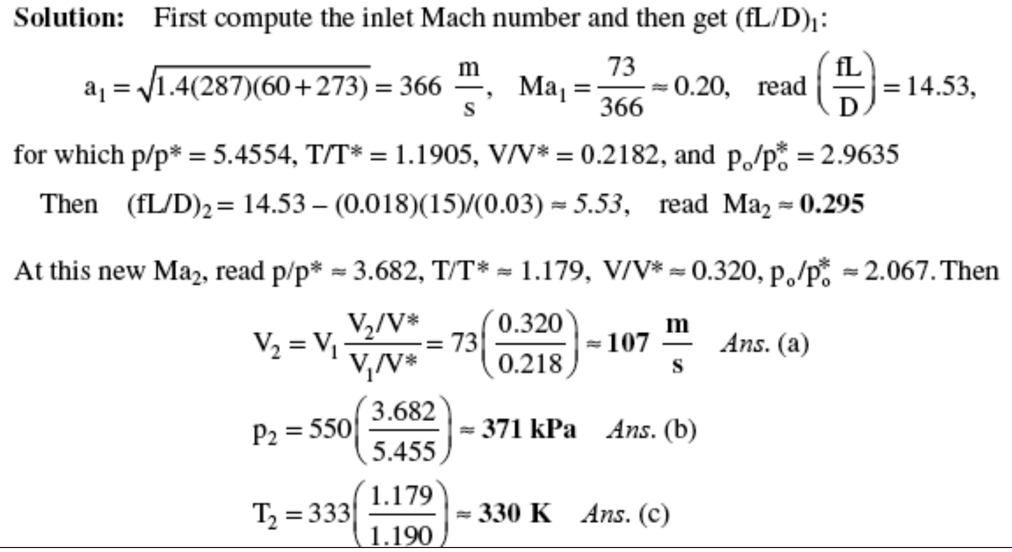
Please note:

1. In the solutions sometimes they use $\overline{f}$ instead of $4f$.
2. The velocity of sound in these solutions is sometimes given the symbol (a) instead of (c).

1

Property	Subsonic flow (M<1)	Supersonic flow (M>1)
Static pressure (p)	Decrease	Increase
Static temperature	Decrease	Increase
Density	Decrease	Increase
Velocity	Increase	Decrease
Mach number	Increase	Decrease
Stagnation pressure	Decrease	Decrease
Stagnation temperature	Constant	Constant
4fl/D	Decrease	Decrease
Entropy (s)	Increase	Increase

2



Solution: First compute the inlet Mach number and then get $(fL/D)_1$:

$$a_1 = \sqrt{1.4(287)(60+273)} = 366\ \frac{m}{s},\quad Ma_1 = \frac{73}{366} \approx 0.20,\quad \text{read}\ \left(\frac{fL}{D}\right) = 14.53,$$

for which $p/p^* = 5.4554$, $T/T^* = 1.1905$, $V/V^* = 0.2182$, and $p_o/p_o^* = 2.9635$

Then $(fL/D)_2 = 14.53 - (0.018)(15)/(0.03) \approx \mathit{5.53}$, read $Ma_2 \approx \mathbf{0.295}$

At this new Ma_2, read $p/p^* \approx 3.682$, $T/T^* \approx 1.179$, $V/V^* \approx 0.320$, $p_o/p_o^* \approx 2.067$. Then

$$V_2 = V_1\frac{V_2/V^*}{V_1/V^*} = 73\left(\frac{0.320}{0.218}\right) \approx \mathbf{107}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}\ \text{(a)}$$

$$p_2 = 550\left(\frac{3.682}{5.455}\right) \approx \mathbf{371\ kPa} \quad \textit{Ans.}\ \text{(b)}$$

$$T_2 = 333\left(\frac{1.179}{1.190}\right) \approx \mathbf{330\ K} \quad \textit{Ans.}\ \text{(c)}$$

	Now we need p_{o1} to get p_{o2}: $p_{o1} = 550[1+0.2(0.2)^2]^{3.5} \approx 566\text{ kPa}, \quad \text{so}\ \ p_{o2} = 566\left(\frac{2.067}{2.964}\right) \approx \mathbf{394\ kPa}$ The extra distance we need to choke the exit to sonic speed is $(fL/D)_2 = 5.53$. That is, $\Delta L = 5.53\frac{D}{f} = 5.53\left(\frac{0.03}{0.018}\right) \approx \mathbf{9.2\ m}$ *Ans.*
3	**Solution:** Noting that $Ma_{exit} = 1.0$, compute Ma_1, find fL/D and hence *f*: $Ma_1 = \frac{V_1}{a_1} = \frac{170}{\sqrt{1.4(287)(300)}} = \frac{170}{347} = 0.49,$ Table B.3: read $f\frac{L}{D} \approx 1.15$ Then $f = \frac{1.15}{40} \approx \mathbf{0.029}$ *Ans.*
4	**Solution:** (a) With no pipe (L = 0), the mass-flow is simply the isentropic maximum: $\dot{m} = \dot{m}_{max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{700000(\pi/4)(0.025)^2}{\sqrt{287(330)}} \approx \mathbf{0.764\ \frac{kg}{s}}$ *Ans.* (a) (b) With a finite length L = 1 m, the flow will choke in the exit plane instead: $Ma_e = 1.0, \quad \frac{fL}{D} = \frac{0.022(1.0)}{0.025} = 0.88, \quad \text{read}\ \ Ma_1(\text{entrance}) \approx 0.525$ Then $T_1 = 330/[1+0.2(0.525)^2] = 313\text{ K}, \quad a_1 = \sqrt{1.4(287)(313)} \approx 354\text{ m/s},$ $V_1 = Ma_1 a_1 = 186\text{ m/s}, \quad p_1 = 700/[1+0.2(0.525)^2]^{3.5} = 580\text{ kPa},$ $\rho_1 = p_1/(RT_1) = 6.46\text{ kg/m}^3$ Finally, then, $\dot{m} = \rho_1 A_1 V_1 = (6.46)(\pi/4)(0.025)^2(186) \approx \mathbf{0.590\ \frac{kg}{s}}$ (23% less) *Ans.* (b) (c) Repeat part (b) for a much longer length, L = 10 m: $\frac{fL}{D} = \frac{0.022(10)}{0.025} = 8.8, \quad Ma_1 = 0.246, \quad T_1 = 326\text{ K}, \quad a_1 = 362\ \frac{m}{s}, \quad V_1 = 89\ \frac{m}{s},$ also, $p_1 = 671\text{ kPa}, \quad \rho_1 = 7.17\ \frac{kg}{m^3}, \quad \dot{m} = \rho_1 A_1 V_1 \approx \mathbf{0.314\ \frac{kg}{s}}$ (59% less) *Ans.* (c)

5	**Solution:** For adiabatic flow, $T^* = \text{constant} = T_o/1.2 = 373/1.2 = 311$ K. The flow chokes in the small exit nozzle, D = 5 cm. Then we estimate Ma_2 from isentropic theory: $\frac{A_2}{A^*} = \left(\frac{6\text{ cm}}{5\text{ cm}}\right)^2 = 1.44$, read $Ma_2(\text{subsonic}) \approx 0.45$, for which $fL/D\rvert_2 \approx 1.52$, $p_2/p^* \approx 2.388$, $p_{o2}/p_o^* \approx 1.449$, $\rho_2/\rho^* \approx 2.070$, $T_2/T^* = 1.153$ or $T_2 \approx 359$ K Given $\dot{m} = 3\ \frac{kg}{s} = \rho_2 A_2 V_2 = \frac{p_2}{287(359)}\left(\frac{\pi}{4}\right)(0.06)^2(0.45)\sqrt{1.4(287)(359)}$, Solve for $p_2 \approx 640$ kPa. Then $p^* = 640/2.388 \approx 268$ kPa At section 1, $\frac{fL}{D} = \frac{fL}{D}\Big\rvert_2 + \frac{f\Delta L}{D} = 1.52 + \frac{0.025(9)}{0.06} \approx 5.27$, read $Ma_1 \approx \mathbf{0.30}$ *Ans.* (a) for which $p_1/p^* \approx 3.6$, or $p_1 \approx 3.6(268) \approx 965$ kPa. Assuming isentropic flow in the inlet nozzle, $p_{tank} \approx 965[1+0.2(0.30)^2]^{3.5} \approx \mathbf{1030\ kPa}$ *Ans.* (b)
6	The Mach numbers are the same, since they depend only upon fL/D (which is the same) and the two nozzle area ratios. If we didn't know the solution to the previous problem, we would guess Ma_1, work out Ma_2 and see if the flow then expands exactly to a sonic exit at the second nozzle. Repeat, if necessary, until the progression through the pipe and the second nozzle is choked. The results are: $Ma_1 = 0.30$, compute $p_1 = 700/[1+0.2(0.30)^2]^{3.5} \approx 658$ kPa. In Table B.3, read $p_1/p^* \approx 3.6$, or $p^* = \frac{658}{3.6} \approx 183$ kPa. Also read $fL/D\rvert_1 \approx 5.27$, subtract $f\Delta L/D$ of 3.75 to find $fL/D\rvert_2 \approx 1.52$, read $Ma_2 \approx \mathbf{0.45}$ *Ans.* (a) Table B.1: $A_2/A^* \approx 1.44$ Then $A_{exit}/A^* = \frac{1.44}{(6/5)^2} \approx 1.0$ (exactly what we want, *sonic flow exit*). Go back to sections 1 or 2 to compute $\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \approx \mathbf{2.04\ kg/s}$ *Ans.* (b)
7	**Solution:** (a) Here the exit pressure *does* equal the receiver pressure: $\frac{p_1}{p^*} = \frac{200}{74} = 2.70$; Table B.3: read $Ma_1 = 0.399$, $f\frac{L}{D} = 2.327$, $\therefore\ L = \frac{2.327(0.05)}{0.02} = \mathbf{5.82\ m}$ *Ans.* (a) (b) $V = Ma_1 a_1 = 0.399\sqrt{1.4(287)(350)} = 150\ \frac{m}{s}$, $\rho_1 = \frac{p_1}{RT_1} = \frac{200000}{287(350)} = 1.99\ \frac{kg}{m^3}$ $\dot{m} = \rho_1 A_1 V_1 = (1.99)\frac{\pi}{4}(0.05)^2(150) = \mathbf{0.585}\ \frac{\mathbf{kg}}{\mathbf{s}}$ *Ans.* (b)

(c) If mass flow increases 50%, and ρ_1 and A_1 are the same, then V_1 and Ma_1 must increase 50%, hence we can immediately calculate the new Mach number:

$$Ma_{1,new} = 1.5(0.399) = 0.599;$$

$$\text{Table B.3:} \quad f\frac{L_{new}}{D} = 0.497, \quad L_{new} = 0.497\frac{0.05}{0.02} = \mathbf{1.24\,m} \quad \textit{Ans.}\ (c)$$

Check in Table B.3 that the exit pressure is $p^*_{new} = 113\text{ kPa} > 74\text{ kPa} = p_{receiver}$.

8 We consider the flow through the converging nozzle to be isentropic and the flow through the constant-area duct to be Fanno flow. A decrease in the pressure at the exit of the constant-area duct (back pressure) causes the mass flowrate through the nozzle and the duct to increase. The flow throughout is subsonic. The maximum flowrate will occur when the back pressure is lowered to the extent that the constant-area duct chokes and the Mach number at the duct exit is equal to 1. Any further decrease of back pressure will not affect the flowrate through the nozzle-duct combination.

For the maximum flowrate condition, the constant-area duct must be choked, and

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{f(\ell_2 - \ell_1)}{D} = \frac{(0.02)(2\text{ m})}{(0.1\text{ m})} = 0.4 \qquad (1)$$

$$Ma_1 = 0.63, \quad \frac{T_1}{T^*} = 1.1, \quad \frac{V_1}{V^*} = 0.66, \quad \frac{p_1}{p^*} = 1.7, \quad \frac{p_{0,1}}{p_0^*} = 1.16$$

Also from isentropic flow tables at M= 0.63, we can get:

$$\frac{T_1}{T_0} = 0.93, \quad \frac{p_1}{p_{0,1}} = 0.76, \quad \frac{\rho_1}{\rho_{0,1}} = 0.83$$

Since T_0 also remains constant through the constant-area duct (see Eq. 11.75), we can use Eq. 11.63 to get T^*. Thus,

$$\frac{T^*}{T_0} = \frac{2}{k+1} = \frac{2}{1.4+1} = 0.8333 \qquad (2)$$

Since $T_0 = 288$ K, we get from Eq. 2,

$$T^* = (0.8333)(288\text{ K}) = 240\text{ K} = T_2 \qquad (3) \quad \text{(Ans)}$$

With T^* known, we can calculate V^* from Eq. 11.36 as

$$\begin{aligned} V^* &= \sqrt{RT^*k} \\ &= \sqrt{[(286.9\text{ J})/(\text{kg}\cdot\text{K})](240\text{ K})(1.4)[1(\text{kg}\cdot\text{m})/(\text{N}\cdot\text{s}^2)][1(\text{N}\cdot\text{m})/\text{J}]} \quad (4) \quad \text{(Ans)} \\ &= 310\text{ m/s} = V_2 \end{aligned}$$

	$V_1 = (0.66)(310\text{ m/s}) = 205\text{ m/s}$ $\rho_1 = 0.83\rho_{0,1} = (0.83)(1.23\text{ kg/m}^3) = 1.02\text{ kg/m}^3$ $\dot{m} = (1.02\text{ kg/m}^3)\left[\dfrac{\pi(0.1\text{ m})^2}{4}\right](206\text{ m/s}) = 1.65\text{ kg/s}$ $T_1 = (0.93)(288\text{ K}) = 268\text{ K}$ $p_1 = (0.76)[101\text{ kPa (abs)}] = 77\text{ kPa (abs)}$ $T_{0,1} = T_{0,2} = 288\text{ K}$ $p_2 = \left(\dfrac{p^*}{p_1}\right)\left(\dfrac{p_1}{p_{0,1}}\right)(p_{0,1}) = \left(\dfrac{1}{1.7}\right)(0.76)[101\text{ kPa(abs)}] = 45\text{ kPa(abs)}$ $p_{0,2} = \left(\dfrac{p_0^*}{p_{0,1}}\right)(p_{0,1}) = \left(\dfrac{1}{1.2}\right)[101\text{ kPa(abs)}] = 84\text{ kPa(abs)}$
9	For a shock to occur as specified, the duct flow must be supersonic. The inlet conditions and nozzle area ratio fix conditions at location 3. For $A_3/A_2 = 2.4$, $M_3 = 2.4$, $p_3/p_{03} = 0.06840$, $p_3/p^* = 0.3111$ and $4fL_{max}/D = 0.4099$. $P^* = P_{01}\,(P_3/P_{01})\,(P^*/P_3) = 800\,(0.0684)\,(1/0.3111) = 175.89$ kPa **But P^* is the same after the shock:** $P_5/P^* = 300/175.89 = 1.705$ → from Fanno flow tables: $M_5=0.62$ From normal shock tables using $M_y=0.62$ → $M_x = M_4 = 1.785$ → $4fL_{max}/D = 0.2365$ $4fL_{duct} / D = 0.4099 - 0.2365 = 0.1734$
10	(a) 1.68, 25.6°; (b) 560 K, 610 kPa.
11	(a) 442.9 m/sec, 768.1 m/sec, 292.3 m/sec, 768.1 m/sec; (b) 343°K, 124.39 kPa; (c) 9.1°.