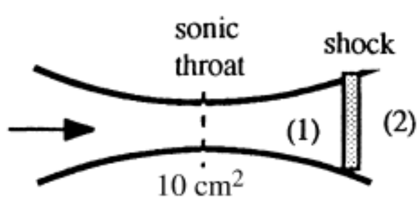
Gas Dynamics

Assignment #3: Normal shock waves

1. Fill the following table for change in flow properties after a normal shock

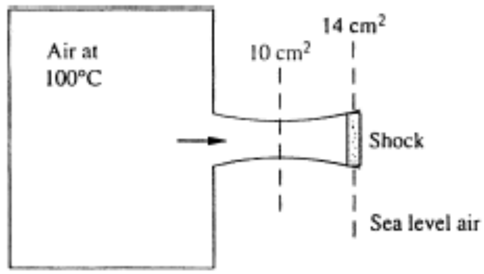
Property	Increase or decrease
Static pressure (p)	Increase
Static temperature	
Density	
Velocity	
Mach number	
Stagnation pressure	
Stagnation temperature	
A^*	
Entropy (s)	

2. Air, supplied by a reservoir at 450 kPa, flows through a converging-diverging nozzle whose throat area is 12 cm^2. A normal shock stands where $A_1 = 20\ cm^2$. (a) Compute the pressure just downstream of this shock. Still farther downstream, where $A_3 = 30$ cm2, estimate (b) p_3; (c) A_3*; and (d) M_3.
3. Air from a reservoir at 20°C and 500 kPa flows through a duct and forms a normal shock downstream of a throat of area 10 cm^2. By an odd coincidence it is found that the stagnation pressure downstream of this shock exactly equals the throat pressure. What is the area where the shock wave stands?



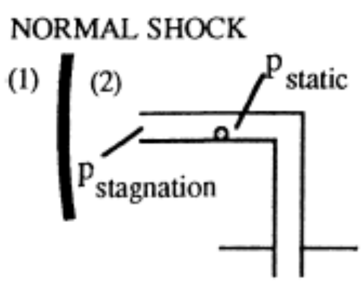
4. Repeat problem #2 except this time let the odd coincidence be that the static pressure downstream of the shock exactly equals the throat pressure. What is the area where the shock wave stands?

5. Air flows from a tank through a nozzle into the standard atmosphere (p = 101.3 kPa), as shown in the figure. A normal shock stands in the exit of the nozzle, as shown. Estimate (a) the tank pressure; and (b) the mass flow.

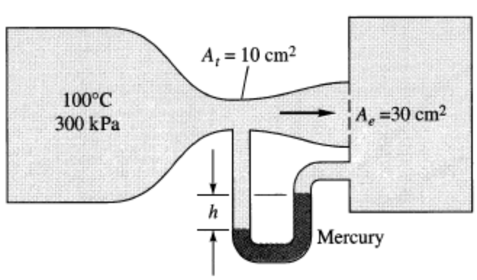


6. Air, at stagnation conditions of 450 K and 250 kPa, flows through a nozzle. At section 1, where the area = 15 cm², there is a normal shock wave. If the mass flow is 0.4 kg/s, estimate (a) the Mach number; and (b) the stagnation pressure just downstream of the shock.

7. When a pitot tube such as in the figure is placed in a supersonic flow, a normal shock will stand in front of the probe. Suppose the probe reads $\mathbf{p_0}$ = 190 kPa and $\mathbf{p}$ = 150 kPa. If the stagnation temperature is 400 K, estimate the (supersonic) Mach number and velocity upstream of the shock.

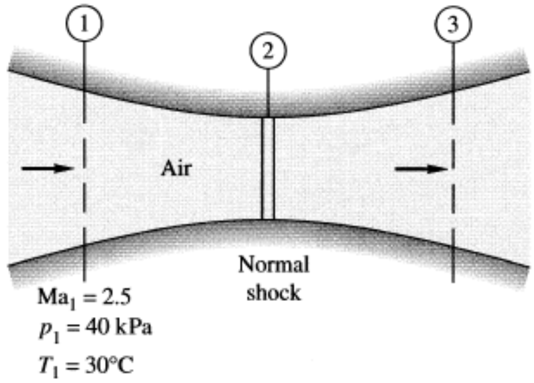


8. Air flows through a converging-diverging nozzle between two large reservoirs, as in the figure. A mercury manometer reads h = 15 cm. Estimate the downstream reservoir pressure. Is there a shock wave in the flow? If so, does it stand in the exit plane or farther upstream?



9. In the previous problem, what would be the mercury manometer reading if the nozzle were operating exactly at supersonic "design" conditions?

10. Air flows through a duct as in the figure, where $\mathbf{A}_1 = 24\ cm^2$, $\mathbf{A}_2 = 18\ cm^2$, and $\mathbf{A}_3 = 32\ cm^2$. A normal shock stands at section 2. Compute (a) the mass flow, (b) the Mach number, and (c) the stagnation pressure at section 3.



Solution of assignment #3

Please note that the velocity of sound in these solutions is sometimes given the symbol (a) instead of (c).

1

Property	Increase or decrease
Static pressure (p)	Increase
Static temperature	Increase
Density	Increase
Velocity	Decrease
Mach number	Decrease
Stagnation pressure	Decrease
Stagnation temperature	Constant
A^*	Increase
Entropy (s)	Increase

2

Solution: If a shock forms, the throat must be **choked** (sonic). Use the area ratio at (1):

$$\frac{A_1}{A^*} = \frac{20}{12} = 1.67, \quad \text{or} \quad Ma_1 \approx 1.985, \quad \text{whence} \quad p_1 = \frac{450}{[1+0.2(1.985)^2]^{3.5}} \approx 59 \text{ kPa}$$

$$\text{Then, across the shock,} \quad \frac{p_2}{p_1} = \frac{2.8(1.985)^2 - 0.4}{2.4} = 4.43,$$

$$p_2 = 4.43(59) \approx \mathbf{261\ kPa} \quad \textit{Ans. (a)}$$

$$\text{Across the shock, at } Ma_1 = 1.985, \quad \frac{A_2^*}{A_1^*} = 1.374, \quad A_2^* = 1.374(12) \approx \mathbf{16.5\ cm^2} \quad \textit{Ans. (c)}$$

$$\text{At } A_3 = 30 \text{ cm}^2, \ \frac{A_3}{A_2^*} = \frac{30}{16.5} = 1.82, \quad \text{whence subsonic } Ma_3 \approx \mathbf{0.34} \quad \textit{Ans. (d)}$$

$$\text{Finally,} \quad p_{o2} = \frac{p_{o1}}{1.374} = 328 \text{ kPa}, \quad p_3 = \frac{328}{[1+0.2(0.34)^2]^{3.5}} \approx \mathbf{303\ kPa} \quad \textit{Ans. (b)}$$

3	**Solution:** If a shock forms, the throat is **sonic**, $A^* = 10\ cm^2$. Now $p_1^* = 0.5283 p_{o1} = 0.5283(500) \approx \mathbf{264\ kPa} = p_{o2}$ also Then $\frac{p_{o2}}{p_{o1}} = \frac{264}{500} = 0.5283$: Table B.2, read $Ma_1 \approx 2.43$ So $A_1/A_1^* = \frac{[1+0.2(2.43)^2]^{3.0}}{1.728(2.43)} \approx 2.47$, or $A_1(\text{at shock}) = 2.47(10) \approx \mathbf{24.7\ cm^2}$ *Ans.*
4	**Solution:** If a shock forms, the throat is **sonic**, $A^* = 10\ cm^2$. Now $p_1^* = 0.5283 p_{o1} = 0.5283(500) = 264\ kPa = p_2$ downstream of the shock Given $p_1 = 500/\left(1+0.2Ma_1^2\right)^{3.5}$ and $p_2/p_1 = \left(2.8Ma_1^2 - 0.4\right)/(2.4)$ and $p_2 = 264$ Solve iteratively for $Ma_1 \approx 2.15$ ($p_1 = 51$ kPa), $A_1/A^* = 1.92$, $\therefore\ \mathbf{A_1 \approx 19.2\ cm^2}$ *Ans.*
5	The throat must be sonic, and the area ratio at the shock gives the Mach number: $A_1/A^* = \frac{14}{10} = 1.4 = \frac{\left[1+0.2Ma_1^2\right]^3}{1.728Ma_1}$, solve $Ma_1 \approx 1.76$ upstream of the shock Then $p_2/p_1\|_{shock} = \frac{2.8(1.76)^2 - 0.4}{2.4} \approx 3.46$, $p_2 = 1$ atm, $p_1 = \frac{101350}{3.46} \approx 29289$ Pa Thus $p_{tank} = p_{o1} = 29289[1+0.2(1.76)^2]^{3.5} \approx \mathbf{159100\ Pa}$ *Ans.* (a) Given that $T_o = 100°C = 373$ K and a critical throat area of $10\ cm^2$, we obtain $\dot{m} = \dot{m}_{max} = 0.6847 p_o A^*/\sqrt{RT_o} = 0.6847(159100)(0.001)/\sqrt{287(373)}$ $\approx \mathbf{0.333\ \frac{kg}{s}}$ *Ans.* (b)
6	**Solution:** If there is a shock wave, then the mass flow is maximum: $\dot{m}_{max} = 0.4\ \frac{kg}{s} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{250000A^*}{\sqrt{287(450)}}$, solve $A^* = 0.000840\ m^2$ Then $\frac{A_1}{A^*} = \frac{0.0015}{0.00084} = 1.786$ Table B.1: Read $Ma_{1,upstream} \approx 2.067$ Finally, from Table B.2, read $\mathbf{Ma_{1,downstream} \approx 0.566}$ *Ans.* (a) Also, Table B.2: $\frac{p_{o2}}{p_{o1}} = 0.690$, $\mathbf{p_{0,downstream}} = 0.690(250) \approx \mathbf{172\ kPa}$ *Ans.* (b)

7	**Solution:** We can immediately find Ma *inside* the shock: $$p_{o2}/p_2 = \frac{190}{150} = 1.267 = \left(1+0.2Ma_2^2\right)^{3.5}, \quad \text{solve } Ma_2 \approx 0.591$$ Then, across the shock, $Ma_1^2 = \frac{0.4(0.591)^2+2}{2.8(0.591)^2-0.4}$, solve $\mathbf{Ma_1 \approx 1.92}$ *Ans.* $$T_1 = \frac{400}{[1+0.2(1.92)^2]} = 230\text{ K}, \quad a_1 = \sqrt{1.4(287)(230)} \approx 304\text{ m/s},$$ $$V_1 = Ma_1 a_1 = (1.92)(304) \approx \mathbf{585\ m/s} \quad \textit{Ans.}$$
8	$$p_{throat} - p_{tank\#2} = (\rho_{mercury} - \rho_{air})gh \approx (13550-0)(9.81)(0.15) \approx 19940\text{ Pa}$$ The lowest possible $p_{throat} = p^* = 0.5283(300) = 158.5$ kPa, for which $p_e \approx 138.5$ kPa But **this** p_e is much lower than would occur in the duct for isentropic subsonic flow. We can check also to see if isentropic *supersonic* flow is a possibility: With $A_e/A^* = 3.0$, the exit Mach number would be 2.64, corresponding to $p_e = 0.047p_o \approx 14$ kPa (?). This is much too low, so that case fails also. Suppose we had supersonic flow with a normal shock wave in the exit plane: $$A_e/A^* = 3.0,\ Ma_e \approx 2.64,\ p_e = 14\text{ kPa},\ \frac{p_{tank\#2}}{p_e} = \frac{2.8(2.64)^2-0.4}{2.4} = 7.95,$$ or: $p_{tank\#2} = 7.95(14) \approx 113$ kPa, compared to p_{tank} (manometer reading) ≈ 138.5 kPa This doesn't match either, the flow expanded too much before the shock wave. Therefore the correct answer is: a **normal shock wave upstream of the exit plane**. *Ans.*
9	Design flow: $\frac{A_e}{A^*} = 3.0, \quad Ma_e = 2.64, \quad p_e = 14\text{ kPa} = p_{tank\ \#2}; \quad p^* = p_{throat} = 158.5\text{ kPa}$ Then $p_t - p_e = 158500 - 14200 = 144300 \approx (13550-0)(9.81)h,$ solve $\mathbf{h \approx 1.09\ m}$ *Ans.*

10	$a_1 = \sqrt{1.4(287)(30+273)} \approx 349\ \text{m/s}, \quad V_1 = 2.5(349) = 872\ \frac{\text{m}}{\text{s}}, \quad \rho_1 = \frac{p_1}{RT_1} = 0.46\ \frac{\text{kg}}{\text{m}^3}$ Then $\dot{m} = \rho_e A_e V_e = 0.46(0.0024)(872) \approx \mathbf{0.96\ kg/s}$ *Ans.* (a) Now move isentropically from 1 to 2 upstream of the shock and thence across to 3: $Ma_1 = 2.5, \quad \therefore \quad \frac{A_1}{A_1^*} = 2.64, \quad A_1^* = \frac{24}{2.64} = 9.1\ \text{cm}^2, \quad \text{and} \quad \frac{A_2}{A_1^*} = \frac{18}{9.1} = 1.98$ Read $Ma_{2,upstream} \approx \mathbf{2.18}, \quad p_{o1} = p_{o2} = 40[1+0.2(2.5)^2]^{3.5} \approx 683\ \text{kPa}$, across the shock, $\frac{A_3^*}{A_2^*} = 1.57, \quad A_3^* = 14.3\ \text{cm}^2, \quad \frac{A_3}{A_3^*} = 2.24\Big\|_{sub}, \quad \mathbf{Ma_3 \approx 0.27}$ *Ans.* (b) Finally, go back and get the stagnation pressure ratio across the shock: at $Ma_2 \approx 2.18, \ \frac{p_{o3}}{p_{o2}} \approx 0.637, \quad \therefore \quad p_{o3} = 0.637(683) \approx \mathbf{435\ kPa}$ *Ans.* (c)