

Gas Dynamics

Assignment #2: Isentropic flow in nozzles

1. Air expands isentropically at 1 kg/s in a converging nozzle with $D_1 = 10$ cm, $p_1 = 150$ kPa, and $T_1 = 100^\circ\text{C}$. The flow discharges to a pressure of 101 kPa. (a) What is the nozzle exit diameter? (b) How much further can the ambient pressure be reduced choking takes place? (Ans. $A_2 = 0.0032$ m², $P_2 = 82.7$ kPa)
2. A bicycle tire is filled with air at 169.12 kPa (abs) and 30°C . The valve breaks, and air exhausts into the atmosphere of 100 kPa (abs) and 20°C . The valve exit is 2-mm diameter and is the smallest area in the system. Assuming one-dimensional isentropic flow, (a) find the initial Mach number, velocity, and temperature at the exit plane. (b) Find the initial mass flow rate. (Ans. $M = 0.9$, $V_e = 291$ m/s, mass flow rate = 0.00122 kg/s)
3. An air tank of volume 1.5 m³ is at 800 kPa and 20°C when it begins exhausting through a converging nozzle to sea-level conditions. The throat area is 0.75 cm². Estimate (a) the initial mass flow; (b) the time to blow down to 500 kPa; and (c) the time when the nozzle ceases being choked. (Ans. Mass flow rate = 0.142 kg/s, $t = 47$ sec., $t = 144$ sec.)
4. Solve problem #1 again using Argon ($k=1.67$, $R=208$ J/kg.K) instead of air. (Ans. $A_2 = 0.0026$ m², $P_2 = 75$ kPa)
5. Consider the converging-diverging nozzle with inlet diameter $D = 5$ cm and throat diameter $d = 3$ cm. Air stagnation temperature is 300 K, and the upstream velocity $V_1 = 72$ m/s. If the throat pressure is 124 kPa, estimate (a) p_1 ; (b) Ma_2 ; and (c) the mass flow. (Ans. $M_2 = 0.831$, $P_1 = 189$ kPa, mass flow rate = 0.313 kg/s)
6. Air, at stagnation conditions of 500 K and 200 kPa, flow through a nozzle. At section 1, where $A = 12$ cm², the density is 0.32 kg/m³. Assuming isentropic flow, (a) find the mass flow. (b) Is the flow choked? If so, estimate A^* . Also estimate (c) p_1 ; and (d) Ma_1 . (Ans. Mass flow rate = 0.257 kg/s, yes the flow is choked, $A^* = 710 \times 10^{-6}$ m², $P_1 = 25.5$ kPa, $M_1 = 2$)
7. Upstream of the throat of an isentropic converging-diverging nozzle at section (1), $V_1 = 150$ m/s, $P_1 = 100$ kPa, and $T_1 = 20$ C, if the discharge flow is supersonic and the throat area is 0.1 m², determine the mass flow rate. Flowing gas is air. (Ans. = 26.5 kg/s)
8. A converging diverging nozzle has entrance and exit areas of 20 cm² and a throat area of 10 cm². If air with stagnation pressure of 100 kPa, flows inside this nozzle, find inlet pressures which can not result in isentropic flow inside the nozzle. (Ans. inlet or exit pressure can not be between 9.396 kPa and 93.71 kPa, i.e. inlet or exit pressure should be either higher than 93.71 kPa, or lower than 9.396 kPa)