Gas Dynamics

Assignment #2: Isentropic flow in nozzles

1. Air expands isentropically at 1 kg/s in a converging nozzle with D_1 = 10 cm, p_1 = 150 kPa, and T_1 = 100°C. The flow discharges to a pressure of 101 kPa. (a) What is the nozzle exit diameter? (b) How much further can the ambient pressure be reduced chocking takes place? (Ans. A_2= 0.0032 m^2, P_2 = 82.7 kPa)
2. A bicycle tire is filled with air at 169.12 kPa (abs) and 30°C. The valve breaks, and air exhausts into the atmosphere of 100 kPa (abs) and 20°C. The valve exit is 2-mm diameter and is the smallest area in the system. Assuming one-dimensional isentropic flow, (a) find the initial Mach number, velocity, and temperature at the exit plane. (b) Find the initial mass flow rate. (Ans. M = 0.9, V_e = 291 m/s, mass flow rate = 0.00122 kg/s)
3. An air tank of volume 1.5 m^3 is at 800 kPa and 20°C when it begins exhausting through a converging nozzle to sea-level conditions. The throat area is 0.75 cm^2. Estimate (a) the initial mass flow; (b) the time to blow down to 500 kPa; and (c) the time when the nozzle ceases being choked. (Ans. Mass flow rate = 0.142 kg/s, t = 47 sec., t = 144 sec.)
4. Solve problem #1 again using Argon (k=1.67, R=208 J/kg.K) instead of air. (Ans. A_2= 0.0026 m^2, P_2 = 75 kPa)
5. Consider the converging-diverging nozzle with inlet diameter **D** = 5 cm and throat diameter **d** = 3 cm. Air stagnation temperature is 300 K, and the upstream velocity $\mathbf{V}_1$ = 72 m/s. If the throat pressure is 124 kPa, estimate (a) $\mathbf{p}_1$; (b) $\mathbf{Ma}_2$; and (c) the mass flow. (Ans. M_2 = 0.831, P_1= 189 kPa, mass flow rate = 0.313 kg/s)
6. Air, at stagnation conditions of 500 K and 200 kPa, flow through a nozzle. At section 1, where A = 12 cm^2, the density is 0.32 kg/m^3. Assuming isentropic flow, (a) find the mass flow. (b) Is the flow choked? If so, estimate A*. Also estimate (c) p_1; and (d) Ma_1. (Ans. Mass flow rate = 0.257 kg/s, yes the flow is chocked, A^*= 710 x 10^{-6} m^2 , P_1 = 25.5 kPa, M_1 = 2)
7. Upstream of the throat of an isentropic converging-diverging nozzle at section (1), V_1 = 150 m/s, P_1 = 100 kPa, and T_1 = 20 C, if the discharge flow is supersonic and the throat area is 0.1 m^2, determine the mass flow rate. Flowing gas is air. (Ans. = 26.5 kg/s)
8. A converging diverging nozzle has entrance and exit areas of 20 cm^2 and a throat area of 10 cm^2. If air with stagnation pressure of 100 kPa, flows inside this nozzle, find inlet pressures which can not result in isentropic flow inside the nozzle. (Ans. inlet or exit pressure can not be between 9.396 kPa and 93.71 kPa, i.e. inlet or exit pressure should be either higher than 93.71 kPa, or lower than 9.396 kPa)

Solution of assignment #2

Please note that the velocity of sound in these solutions is sometimes given the symbol (a) instead of (c).

1

a) $\rho_1 = \frac{P}{RT_1} = \frac{150\times10^3}{287\times373} = 1.4 \ kg/m^3$

$V_1 = \frac{\dot{m}}{\rho_1 A_1} = \frac{1}{1.4\times\frac{\pi}{4}(0.1)^2} = 90.87 \ m/s$

$M_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{90.87}{\sqrt{1.4\times287\times373}} = \boxed{0.235}$

from tables $\frac{A_1}{A^*} = 2.54$, $\frac{P_1}{P_o} = 0.962$

$\frac{P_2}{P_o} = P_2 \frac{P_1/P_o}{P_1}$

$= 101 \times \frac{0.962}{150} = 0.648$

$\therefore$ from tables $\boxed{M_2 = 0.81}$, $A_2/A^* = 1.0342$

$\therefore A_2 = A_1 \frac{A_2/A^*}{A_1/A^*}$

$= \frac{\pi}{4}(0.1)^2 \frac{1.0342}{2.54} = \boxed{3.2\times10^{-3} \ m^2}$

b) for chocking $M_2 = 1$

$\therefore \frac{P_2}{P_o} = 0.52828$

$\therefore P_2 = P_1 \frac{P_2/P_o}{P_1/P_o}$

$= 150\times\frac{0.52828}{0.962} = \boxed{82.37 \ kPa}$

$\therefore$ The ambient pressure can be reduced to 82.37 kPa before chocking takes place.

2	**Solution:** (a) Flow is *not* choked, because the pressure ratio is less than 1.89: $$\frac{p_o}{p}=\frac{169.12}{100}=\left(1+0.2Ma_e^2\right)^{3.5},\quad \text{solve } \mathbf{Ma_e = 0.90};\quad \text{Read } T_e = 0.8606T_o = \mathbf{261\ K}$$ $$V_e = Ma_e a_e = (0.90)\sqrt{1.4(287)(261)} = 0.90(324) = \mathbf{291\ \frac{m}{s}} \quad \textit{Ans. (a)}$$ (b) Evaluate the exit density at Ma = 0.90 and thence the mass flow: $$\rho_e=\frac{p_e}{RT_e}=\frac{100000}{287(261)}=1.335\ \frac{kg}{m^3},$$ $$\textit{Then}\quad \dot{m}=\rho_e A_e V_e=(1.335)\frac{\pi}{4}(0.002)^2(291)=\mathbf{0.00122\ \frac{kg}{s}}\quad \textit{Ans. (b)}$$
3	**Solution:** For sea level, $p_{ambient}$ = 101.35 kPa < 0.528p_{tank}, hence the flow is choked until the tank pressure drops to $p_{ambient}$/0.528 = 192 kPa. (a) We obtain $$\dot{m}_{initial}=\dot{m}_{max}=0.6847\frac{p_o A^*}{\sqrt{RT_o}}=0.6847\frac{800000(0.75E\text{–}4\ m^2)}{\sqrt{287(293)}}=\mathbf{0.142}\ \frac{kg}{s}\quad \textit{Ans. (a)}$$ (b) For a control volume surrounding the tank, a mass balance gives $$\frac{d}{dt}(\rho_o\upsilon)=\frac{\upsilon}{RT_o}\frac{dp_o}{dt}=-\dot{m}=-0.6847\frac{p_o A^*}{\sqrt{RT_o}},\quad \text{separate the variables:}$$ $$\frac{p(t)}{p(0)}=\exp\left[-0.6847\frac{A^*\sqrt{RT_o}}{\upsilon}t\right]=\mathbf{e}^{-0.00993t}\quad \text{until p(t) drops to 192 kPa}$$ At 500 kPa, we obtain 500/800 = exp(–0.00993t), or **t ≈ 47 s** *Ans.* (b) At choking (192 kPa), 192/800 = exp(–0.00993t), or **t ≈ 144 s** *Ans.* (c)

4	**Solution:** For argon, from Table A.4, $R = 208$ J/kg·K and $k = 1.67$. $$\rho_1 = \frac{150000}{208(373)} = 1.93\ \frac{\text{kg}}{\text{m}^3},\quad \dot{m} = 1\ \frac{\text{kg}}{\text{s}} = 1.93\frac{\pi}{4}(0.1)^2 V_1,\quad \therefore\quad V_1 = 66\ \frac{\text{m}}{\text{s}}$$ $$Ma_1 = \frac{66}{\sqrt{1.67(208)(373)}} = 0.183,\quad \frac{A_1}{A^*} = \frac{1}{0.183}\left[\frac{1+0.335(0.183)^2}{(1+1.67)/2}\right]^{\frac{1.67+1}{2(1.67-1)}} = 3.14$$ $$p_o = 150[1+0.335(0.183)^2]^{\frac{1.67}{0.67}} = 154\,\text{kPa},$$ $$\frac{p_e}{p_o} = \frac{101}{154} = \left(1+0.335\,Ma_e^2\right)^{\frac{-1.67}{0.67}},\quad \mathbf{Ma_e = 0.743}$$ $A_2/A^*=1.06$ $A_2 = (\pi/4) * 0.1^2 * (1.06/3.21) = 0.0026\ \text{m}^2$ Thus the exit flow is *not* choked. We could decrease the ambient pressure to **75 kPa** before the flow would choke. The maximum mass flow is about 1.01 kg/s.
5	$$T_o = T_1 + \frac{V_1^2}{2c_p} = 300\,\text{K} = T_1 + \frac{(72\ \text{m/s})^2}{2(1005\,\text{J/kg·K})},\quad \text{solve for } T_1 = 297.4\,\text{K}$$ $$Ma_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{72}{\sqrt{1.4(287)(297.4)}} = \frac{72\,\text{m/s}}{346\,\text{m/s}} = 0.208$$ Area-ratio calculations will then yield A^* and Ma_2 and then p_o and p_1: $$\frac{A_1}{A^*} = \frac{(\pi/4)(0.05\,\text{m})^2}{A^*} = \frac{\left(1+0.2\,Ma_1^2\right)^3}{1.728\,Ma_1} = \frac{[1+0.2(0.208)^2]^3}{1.728(0.208)} = 2.85,$$ $$\text{Solve}\quad A^* = 0.0006886\ \text{m}^2$$ $$\frac{A_2}{A^*} = \frac{(\pi/4)(0.03\ \text{m})^2}{0.0006886\ \text{m}^2} = \frac{\left(1+0.2Ma_2^2\right)^3}{1.728Ma_2} = 1.027,\quad \text{Solve}\ \ \mathbf{Ma_2 = 0.831}\quad \textit{Ans. (b)}$$ $$p_o = p_2\left(1+0.2\,Ma_2^2\right)^{3.5} = (124\,\text{kPa})[1+0.2(0.831)^2]^{3.5} = 195\,\text{kPa}$$ $$p_1 = p_o \Big/ \left(1+0.2\,Ma_1^2\right)^{3.5} = (195\,\text{kPa})/[1+0.2(0.208)^2]^{3.5} = \mathbf{189\ kPa}\quad \textit{Ans. (a)}$$ The mass flow follows from any of several formulas. For example: $$\dot{m} = \rho_1 A_1 V_1 = \left(\frac{p_1}{RT_1}\right)A_1V_1 = \left[\frac{189000}{287(297.4)}\right]\left(\frac{\pi}{4}\right)(0.05)^2(72) = \mathbf{0.313}\ \frac{\mathbf{kg}}{\mathbf{s}}\quad \textit{Ans. (c)}$$

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Solution: Evaluate stagnation density, density ratio, and Mach number:

$$\rho_o = \frac{p_o}{RT_o} = \frac{200000}{287(500)} = 1.39\ \frac{\text{kg}}{\text{m}^3};$$

$$\frac{\rho_o}{\rho} = \frac{1.39}{0.32} = \left(1+0.2\,Ma_1^2\right)^{2.5},\quad \text{solve}\ \mathbf{Ma_1 = 2.00}\quad \textit{Ans.}\ \text{(d)}$$

$$T_1 = 500/[1+0.2(2.00)^2] = 278\ \text{K},\quad V_1 = Ma_1 a_1 = 2.00[1.4(287)(278)]^{1/2} = 668\ \frac{\text{m}}{\text{s}}$$

$$\text{Finally,}\quad \dot{m} = \rho_1 A_1 V_1 = 0.32(12E\text{–}4)(668) = \mathbf{0.257\ \frac{kg}{s}}\quad \textit{Ans.}\ \text{(a)}$$

The **flow is clearly choked**, because Ma_1 is supersonic. A throat exists:

$$\dot{m} = 0.257 = \dot{m}_{max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{200000\,A^*}{\sqrt{287(500)}},$$

$$\text{solve}\quad \mathbf{A^* = 0.000710\ m^2}\quad \textit{Ans.}\ \text{(b)}$$

(c) Also calculate

$$p_1 = \frac{p_o}{\left(1+0.2\,Ma_1^2\right)^{3.5}} = \frac{200000}{[1+0.2(2.00)^2]^{3.5}} = 25500\ \text{Pa}\quad \textit{Ans.}\ \text{(c)}$$

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$$P_o = \frac{100\ \text{kPa(abs)}}{0.87} = 115\ \text{kPa(abs)}$$

and with Eq. 10

$$p^* = [115\ \text{kPa(abs)}](0.52828) = 60.8\ \text{kPa(abs)}$$

Then with Eq. 8

$$\rho^* = \frac{\left(60.8\times10^3\ \frac{N}{m^2}\right)}{\left(286.9\ \frac{N.m}{kg.K}\right)(254\ K)} = 0.83\ \frac{kg}{m^3}$$

Finally, with Eq. 2 we obtain

$$\dot{m} = \left(0.83\ \frac{kg}{m^3}\right)(0.1\ m^2)\left(319\ \frac{m}{s}\right) = \underline{\underline{26.5}}\ \frac{kg}{s}$$

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$$Ma_1 = \frac{\left(150\ \frac{m}{s}\right)}{\sqrt{\dfrac{\left(286.9\ \frac{N.m}{kg.K}\right)(293K)(1.4)}{\left(1\ \frac{N}{kg.\frac{m}{s^2}}\right)}}} = 0.4372$$

Thus, the flow is choked at the throat. From Eq. 7 we obtain for corresponding value in Fig.D.1 for $Ma_1 = 0.44$

$$T_o = \frac{293\ K}{(0.96)} = 305\ K$$

With Eq. 5 we obtain

$$T^* = (305\ K)(0.83333) = 254\ K$$

Thus

$$V^* = \sqrt{\frac{\left(286.9\ \frac{N.m}{kg.K}\right)(254\ K)(1.4)}{\left(1\ \frac{N}{kg.\frac{m}{s^2}}\right)}} = 319\ \frac{m}{s}$$

8

Solution: There is a subsonic entrance region of high pressure and a supersonic entrance region of low pressure, both of which are bounded by a sonic (critical) throat, and both of which have a ratio $A_{x=0}/A^* = 2.0$. From Table B.1 or Eq. (9.44), we find these two conditions to be bounded by

a) subsonic entrance: $A/A^* = 2.0$, $Ma_e \approx 0.306$, $p_e \approx 0.9371 p_o \approx$ **93.71 kPa**

b) supersonic entrance: $A/A^* = 2.0$, $Ma_e \approx 2.197$, $p_e \approx 0.09396 p_o \approx$ **9.396 kPa**

Thus *no isentropic flow can exist* between entrance pressures $9.396 < p_e < 93.71$ kPa. The complete family of isentropic pressure curves is shown in the graph on the following page. They are **not** easy to find, because we have to convert implicitly from area ratio to Mach number.