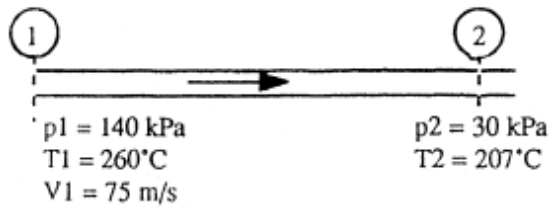
Gas Dynamics

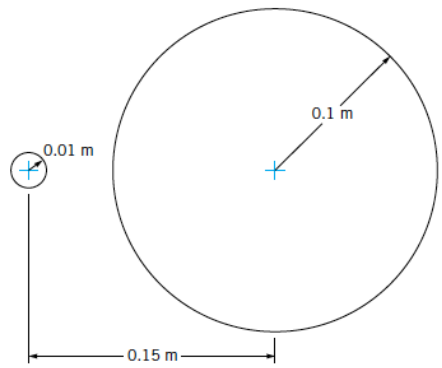
Assignment #1: Speed of sound

1. An ideal gas flows adiabatically through a duct. At section 1, p1 = 140 kPa, T1 = 260°C, and V1 = 75 m/s. Farther downstream, p2 = 30 kPa and T2 = 207°C. Calculate V2 in m/s if the gas is (a) air, k = 1.4, and (b) argon, k = 1.67.

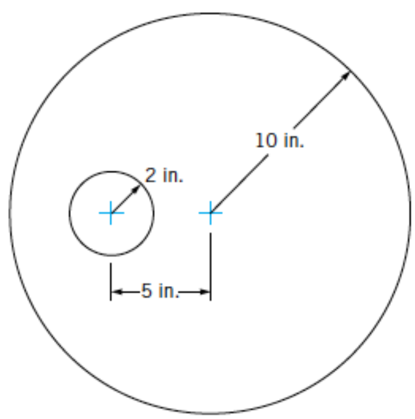


2. Helium (k=1.66, R=2077 j/kg.K) at 300°C and 200 kPa, in a closed container, is cooled to a pressure of 100 kPa. Estimate the new temperature, in °C.
3. A certain aircraft flies at the same Mach number regardless of its altitude. Compared to its speed at 12000-m Standard Altitude (T=216 K), it flies 127 km/h faster at sea level (T=288 K). Determine its Mach number.
4. At 300°C and 1 atm, estimate the speed of sound of (a) nitrogen (k=1.4, R=297); (b) hydrogen (k=1.41, R=4124); (c) helium (k=1.66, R=2077 J/kg.K); (d) steam (k=1.33, R=461); and (e) uranium hexafluoride 238UF6 (k = 1.06, molecular weight=352).
5. A weak pressure wave (sound wave), with a pressure change $\Delta p \approx 40$ Pa, propagates through still air at 20°C and 1 atm. Estimate (a) the density change; (b) the temperature change; and (c) the velocity change across the wave. Use the relations we derived in lecture #1 ($\Delta p \approx \rho C \Delta V$, $C\, d\rho = \rho\, dV$).
6. The Concorde aircraft flies at Ma ≈ 2.3 at 11-km standard altitude (T=216 K). Estimate the temperature in °C at the front stagnation point. At what Mach number would it have a front stagnation point temperature of 450°C?
7. A gas flows at V = 200 m/s, p = 125 kPa, and T = 200°C. For (a) air and (b) helium, compute the maximum pressure and the maximum velocity attainable by expansion or compression.
8. CO2 (k=1.3, R=189 j/kg.K) expands isentropically through a duct from p1 = 125 kPa and T1 = 100°C to p2 = 80 kPa and V2 = 325 m/s. Compute (a) T2; (b) M2; (c) To; (d) p_0; (e) V1; and (f) M1.

9. At a given instant of time, two pressure waves, each moving at the speed of sound, emitted by a point source moving with constant velocity in a fluid at rest are shown in the Figure. Determine the Mach number involved and indicate with a sketch the instantaneous location of the point source.



10. At a given instant of time, two pressure waves, each moving at the speed of sound, emitted by a point source moving with constant velocity in a fluid at rest, are shown in the Figure. Determine the Mach number involved and indicate with a sketch the instantaneous location of the point source.



11. List how the following variables change with the increase in cross section area at subsonic and supersonic flows: pressure, temperature, velocity, density, and Mach number.

Solution of assignment #1

Please note that the velocity of sound in these solutions is sometimes given the symbol (a) instead of (c).

1	**Solution:** (a) For air, take $k = 1.40$, $R = 287$ J/kg·K, and $c_p = 1005$ J/kg·K. The adiabatic steady-flow energy equation (9.23) is used to compute the downstream velocity: $$c_pT+\frac{1}{2}V^2= \text{constant} = 1005(260)+\frac{1}{2}(75)^2= 1005(207)+\frac{1}{2}V_2^2 \quad \text{or} \quad \mathbf{V_2 \approx 335\ \frac{m}{s}} \quad \textit{Ans.}$$ (b) For argon, take $k = 1.67$, $R = 208$ J/kg·K, and $c_p = 518$ J/kg·K. Repeat part (a): $$c_pT+\frac{1}{2}V^2 = 518(260)+\frac{1}{2}(75)^2 = 518(207)+\frac{1}{2}V_2^2, \quad \text{solve} \quad \mathbf{V_2 = 246\ \frac{m}{s}} \quad \textit{Ans.}$$
2	**Solution:** From Table A.4 for helium, $k = 1.66$ and $R = 2077\ m^2/s^2\cdot K$. Convert 300°C to 573 K. (a) The density is unchanged because the container is constant volume. Thus $$\frac{p_2}{p_1}=\frac{100\text{ kPa}}{200\text{ kPa}}=\frac{\rho_2 RT_2}{\rho_1 RT_1}=\frac{T_2}{T_1}=\frac{T_2}{573\ K}, \quad \text{solve for } T_2 = 287\ K = \mathbf{14°C} \quad \textit{Ans.}\text{ (a)}$$
3	**Solution:** At sea level, $T_1 = 288.16$ K. At 12000 m standard, $T_2 = 216.66$ K. Then $$a_1 = \sqrt{kRT_1} = \sqrt{1.4(287)(288.16)} = 340.3\ \frac{m}{s}; \quad a_2 = \sqrt{kRT_2} = 295.0\ \frac{m}{s}$$ Then $$\Delta V_{plane} = Ma(a_2 - a_1) = Ma(340.3 - 295.0) = [127\text{ km/h}] = 35.27\text{ m/s}$$ Solve for $$\mathbf{Ma} = \frac{35.27}{45.22} \approx \mathbf{0.78} \quad \textit{Ans.}$$
4	**Solution:** The gas constants are listed in Appendix Table A.4 for all but uranium gas (e): (a) nitrogen: $k = 1.40$, $R = 297$, $T = 300 + 273 = 573$ K: $$a = \sqrt{kRT} = \sqrt{1.40(297)(573)} \approx \mathbf{488\ m/s} \quad \textit{Ans.}\text{ (a)}$$ (b) hydrogen: $k = 1.41$, $R = 4124$, $a = \sqrt{1.41(4124)(573)} \approx \mathbf{1825\ m/s}$ *Ans.* (b) (c) helium: $k = 1.66$, $R = 2077$: $a = \sqrt{1.66(2077)(573)} \approx \mathbf{1406\ m/s}$ *Ans.* (c)

	(d) steam: $k = 1.33, R = 461: \quad a = \sqrt{1.33(461)(573)} \approx \mathbf{593\ m/s}$ *Ans.* (d) (e) For uranium hexafluoride, we need only to compute *R* from the molecular weight: (e) $^{238}UF_6: \quad M = 238 + 6(19) = 352, \quad \therefore\ R = \frac{8314}{352} \approx 23.62\ m^2/s^2 \cdot K$ then $a = \sqrt{1.06(23.62)(573)} \approx \mathbf{120\ m/s}$ *Ans.* (e)
5	**Solution:** For air at 20°C, speed of sound $a \approx 343$ m/s, and $\rho = 1.2\ kg/m^3$. Then $\Delta p \approx \rho C \Delta V, C \approx a, \quad \text{thus } 40 = (1.2)(343)\Delta V, \quad \text{solve for } \Delta V \approx \mathbf{0.097\ \frac{m}{s}}$ *Ans.* (a) $\Delta\rho = (\rho + \Delta\rho)\frac{\Delta V}{C} = (1.2 + \Delta\rho)\frac{0.097}{343}, \quad \text{solve for } \Delta\rho \approx \mathbf{0.00034\ kg/m^3}$ *Ans.* (b) $\frac{T + \Delta T}{T} \approx \left(\frac{p + \Delta p}{p}\right)^{(k-1)/k}, \text{ or: } \frac{293 + \Delta T}{293} \approx \left(\frac{101350 + 40}{101350}\right)^{\frac{0.4}{1.4}}, \quad \Delta T \approx \mathbf{0.033\ K}$ *Ans.* (c)
6	**Solution:** At 11-km standard altitude, $T \approx 216.66$ K, $a = \sqrt{(kRT)} = 295$ m/s. Then $T_{nose} = T_o = T\left(1 + \frac{k-1}{2}Ma^2\right) = 216.66[1 + 0.2(2.3)^2] = 446\ K \approx \mathbf{173°C}$ *Ans.* If, instead, $T_o = 450°C = 723\ K = 216.66(1 + 0.2\ Ma^2)$, solve $Ma \approx \mathbf{3.42}$ *Ans.*
7	**Solution:** Given (V, p, T), we can compute Ma, T_o and p_o and then $V_{max} = \sqrt{(2c_p T_o)}$: (a) air: $Ma = \frac{V}{\sqrt{kRT}} = \frac{200}{\sqrt{1.4(287)(200 + 273)}} = \frac{200}{436} = 0.459$ Then $p_{max} = p_o = p\left(1 + \frac{k-1}{2}Ma^2\right)^{k/(k-1)} = 125[1 + 0.2(0.459)^2]^{3.5} \approx \mathbf{144\ kPa}$ *Ans.* (a) $T_o = (200 + 273)[1 + 0.2(0.459)^2] = 493\ K, \quad V_{max} = \sqrt{2(1005)(493)} \approx \mathbf{995\ m/s}$ *Ans.* (a) (b) For helium, $k = 1.66$, $R = 2077\ m^2/s^2 \cdot K$, $c_p = kR/(k - 1) = 5224\ m^2/s^2 \cdot K$. Then $Ma = 200/\sqrt{1.66(2077)(473)} \approx 0.157, \quad p_o = 125[1 + 0.33(0.157)^2]^{\frac{1.66}{0.66}} \approx \mathbf{128\ kPa}$ $T_o = 473[1 + 0.33(0.157)^2] = 477\ K, \quad V_{max} = \sqrt{2(5224)(477)} \approx \mathbf{2230\ m/s}$ *Ans.* (b)

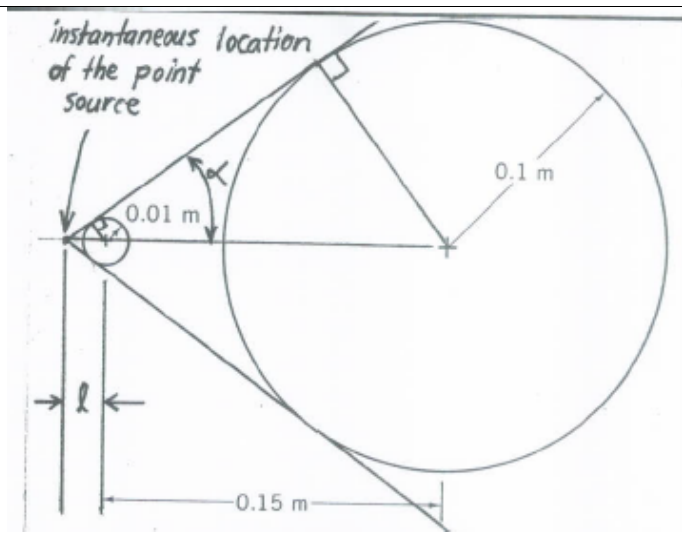
8	**Solution:** For CO_2, from Table A.4, take $k = 1.30$ and $R = 189$ J/kg·K. Compute the specific heat: $c_p = kR/(k-1) = 1.3(189)/(1.3-1) = 819$ J/kg·K. The results follow in sequence: (a) $T_2 = T_1(p_2/p_1)^{(k-1)/k} = (373\text{ K})(80/125)^{(1.3-1)/1.3} = \mathbf{336\text{ K}}$ *Ans.* (a) (b) $a_2 = \sqrt{kRT_2} = \sqrt{(1.3)(189)(336)} = 288\text{ m/s},\ Ma_2 = V_2/a_2 = 325/288 = \mathbf{1.13}$ *Ans.* (b) (c) $T_{o1} = T_{o2} = T_2\left(1+\frac{k-1}{2}Ma_2^2\right) = (336)\left[1+\frac{0.3}{2}(1.13)^2\right] = \mathbf{401\text{ K}}$ *Ans.* (c) (d) $p_{o1} = p_{o2} = p_2\left(1+\frac{k-1}{2}Ma_2^2\right)^{1.3/(1.3-1)} = (80)\left[1+\frac{0.3}{2}(1.13)^2\right]^{1.3/0.3} = \mathbf{171\text{ kPa}}$ *Ans.* (d) (e) $T_{o1} = 401\ K = T_1 + \frac{V_1^2}{2c_p} = 373 + \frac{V_1^2}{2(819)}$, Solve for $\mathbf{V_1 = 214\text{ m/s}}$ *Ans.* (e) (f) $a_1 = \sqrt{kRT_1} = \sqrt{(1.3)(189)(373)} = 303\text{ m/s},\ Ma_1 = V_1/a_1 = 214/303 = \mathbf{0.71}$ *Ans.* (f)
9	

FIGURE P11.23

The Mach number associated with the motion of the point source involved in the sketch above is easily obtained with Eq. 11.39 as shown below.

$$Ma = \frac{1}{\sin\alpha}$$

From the sketch above we note that

$$\sin\alpha = \frac{0.01\,m}{\ell} = \frac{0.1\,m}{0.15\,m + \ell}$$

Thus

$$(0.01\,m)(0.15\,m + \ell) = (0.1\,m)\,\ell$$

or

$$\ell = \frac{(0.01\,m)(0.15\,m)}{(0.09\,m)} = \underline{\underline{0.0167\,m}}$$

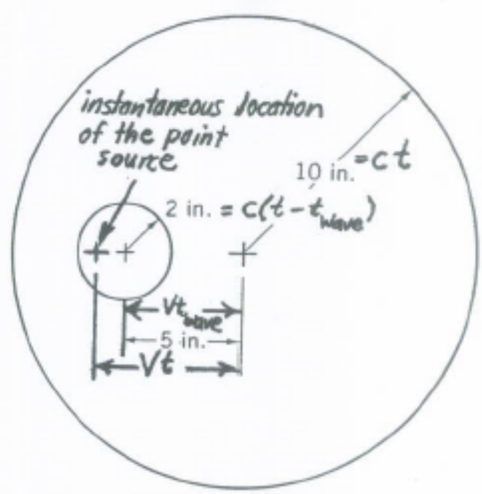
and

$$\sin\alpha = \frac{0.01\,m}{0.0167\,m} = 0.599$$

Thus

$$Ma = \frac{1}{\sin\alpha} = \frac{1}{0.599} = \underline{\underline{1.67}}$$

10



To determine the Mach number, Ma, we use

$$Ma = \frac{V t_{wave}}{c\, t_{wave}} \qquad (1)$$

However, from the sketch above we have

$$c(t - t_{wave}) = 2\text{ in.} = ct - ct_{wave} = 10\text{ in.} - ct_{wave}$$

Thus,

$$ct_{wave} = 10\text{ in.} - 2\text{ in.} = 8\text{ in.}$$

and with Eq. 1

$$Ma = \frac{5\text{ in.}}{8\text{ in.}} = \underline{\underline{0.625}}$$

Also

$$Ma = \frac{vt}{ct} = \frac{vt}{10\text{ in.}} = 0.625$$

Thus,

$$vt = (0.625)(10\text{ in.}) = \underline{\underline{6.25\text{ in.}}}$$

11 For a duct with increasing cross section area:

Property	Subsonic (M<1)	Supersonic (M>1)
Pressure	Increase	Decrease
Temperature	Increase	Decrease
Density	Increase	Decrease
Velocity	Decrease	Increase
Mach number	Decrease	Increase